



# Precision cosmological measurements: Independent evidence for dark energy

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## Abstract

Using recent precision measurements of cosmological parameters, we re-examine whether these observations alone, independent of type Ia supernova surveys, are sufficient to imply the existence of dark energy. We find that best measurements of the age of the Universe  $t_0$ , the Hubble parameter  $H_0$  and the matter fraction  $\Omega_m$  strongly favor an equation of state defined by ( $w < -1/3$ ). This result is consistent with the existence of a repulsive, acceleration-causing component of energy if the Universe is nearly flat.

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The current era in cosmology seems to be the first in which local astrophysical measurements are consistent with the generally accepted large scale cosmology. To provide some historical context, consider the period from 1980 to roughly 1995. Inflation offered us a large scale model for cosmology, requiring  $\Omega_{\text{total}} = 1$ , which could not find verification in measurements on smaller scales. Attempts to dynamically determine  $\Omega_{\text{total}}$  (e.g., [1,2]) consistently returned results of  $\Omega_{\text{total}} \sim 0.25 \pm 0.10$ . This led to the notion [3] that, under the  $\Omega_{\text{total}} = 1$  prior, there must be a bias between the distribution of light (e.g., galaxies) and mass (e.g., the dark matter component). Not only did the Universe have to be dark matter dominated, the distribution of that dark matter had to be significantly different than the distribution of light. At the time, this was the only way to reconcile the small scale measurements with the large scale (inflation) requirement.

In this Letter we reinvestigate whether recent determinations of cosmological parameters are sufficient, by themselves, to imply the existence of dark energy—specifically, a component of energy with equation of state  $w \equiv p/\rho < -1/3$ . In the mid-90's

several authors [4,5] analyzed aggregate data based on globular cluster ages, clustering of galaxies, Big Bang nucleosynthesis, and the Hubble constant and concluded that something like a cosmological constant might be necessary to produce a flat Universe. However, the conclusions were not definitive at the time due to the large uncertainty in the observational parameters. Our purpose is to update these earlier investigations, accounting for improvements in precision. We will argue that observations of key parameters such as the age of the Universe  $t_0$ , the Hubble parameter  $H_0$  and the matter fraction  $\Omega_m$  have become definitive in support of dark energy. One might question the need for this analysis in the post-WMAP era, but it is important to understand whether increasingly precise measurements are consistent with the concordance cosmology obtained from best fits of WMAP data. Indeed, given the dramatic nature and consequences of dark energy, it is important to understand the observational evidence for it as broadly and robustly as possible.

Despite the impressive results of the type Ia supernova collaborations [6], it is still possible that dust [7], evolution effects [8] or exotic particle physics [9] might alter the interpretation of the extracted redshift-distance relation. For example, the axion models in [9] account for the dimness of distant supernovae by conversion of photons into axions in background galactic magnetic fields, rather than through accelerated expansion. Exotic

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particle physics models which are less well motivated than axions, but perhaps no more counterintuitive than the existence of dark energy itself, might in principle explain the supernova data without requiring acceleration. However, the demonstration that a dominant component of energy with  $w \equiv p/\rho < -1/3$  is strongly favored by the observed values of cosmological parameters provides a direct and robust argument for acceleration.

We seek evidence for a component which has equation of state  $w \equiv p/\rho < -1/3$ . Recall the Einstein equation

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3} \sum_i (\rho_i + 3p_i). \quad (1)$$

The sign of the acceleration  $\ddot{R}$  is determined by the sign of  $\sum_i (\rho_i + 3p_i)$ , where the sum runs over all contributions to the energy momentum tensor. Strictly speaking,  $w < -1/3$  is the threshold for a component to cause acceleration when it is the only form of energy. If other forms of energy are non-negligible the overall sign of the right-hand side of (1) might still be negative (i.e., the Universe is decelerating, albeit more slowly than otherwise) even in the presence of energy with  $w < -1/3$ . Asymptotically, though, the component with the smallest positive or most negative value of  $w$  will eventually dominate all others. We recall that a cosmological constant has  $w = -1$ , while a dynamical scalar model with non-zero vacuum energy typically has  $-1 < w < 0$ . Values of  $w$  less than  $-1$  violate the null energy condition, and are generally associated with instabilities [10].

Analysis of the 3 year WMAP data [11] favors a negative pressure equation of state for models with constant  $w$  when constraints on the matter energy density are included (i.e., from the Sloan Digital Sky Survey or the 2dF Galaxy Redshift Survey). In this Letter we conduct a simpler analysis in which the priors are transparent and easy to state.

We find that best measurements of the age of the Universe  $t_0$ , the Hubble parameter  $H_0$  and the matter fraction  $\Omega_m$  are sufficient to require the existence, during some cosmologically significant epoch, of a repulsive, acceleration-causing ( $w < -1/3$ ) component of energy, assuming the Universe is nearly flat. A relation between these quantities is obtained using Einstein's equation for a Friedmann–Robertson–Walker Universe. The analysis itself is not necessarily new, but it can now be applied for the very first time with stringent constraints due to recent precision measurements of the relevant cosmological parameters.

The age of the Universe is given by

$$t_0 = \int_0^{R(t_0)} \frac{dR}{\dot{R}}, \quad (2)$$

which yields

$$t_0 H_0 = \int_0^1 \frac{dx}{(\Omega_m x^{-1} + \Omega_{de} x^{-1-3w})^{1/2}}, \quad (3)$$

where we have taken  $w$  constant in time and neglected the radiation component as it is numerically small. We also assume

flatness, which implies  $\Omega_{de} = 1 - \Omega_m$ , and allows us to define the integral as  $I(\Omega_m, w)$ . The quantities  $t_0$ ,  $H_0$  and  $\Omega_m$  then determine  $w$ .

In the more general case, where the dark energy component has time varying equation of state  $w(t)$ , the second term in the denominator of the integral in (3) (the dark energy term) is more complicated, having the form

$$\Omega_{de} \exp \left[ \int_x^1 \frac{dx'}{x'} (1 + 3w(x')) \right]. \quad (4)$$

If  $(1 + 3w(x')) > 0$  for all  $x < x' < 1$ , the dark energy term (4) is always decreasing with increasing  $x$ , and the denominator in (3) is larger for all  $x$  than it would be in the special case  $w = -1/3$ , where (4) is constant. Therefore, if the dark energy *never* exhibits a repulsive equation of state, so  $w(t) > -1/3$  at all times, the integral is bounded above:

$$I(\Omega_m, w > -1/3) < I(\Omega_m, -1/3). \quad (5)$$

Similarly, we deduce

$$I(\Omega_m, w > w^*) < I(\Omega_m, w^*). \quad (6)$$

In other words, in the most general case, unless the dark energy behaved repulsively during some earlier epoch, the integral  $I$ , and hence the product  $t_0 H_0$ , is bounded above by  $I(\Omega_m, -1/3)$ . Using measured values of  $t_0$ ,  $H_0$  and  $\Omega_m$ , it is therefore possible to deduce that a repulsive epoch must have occurred. (Note an epoch with repulsive energy does not necessarily imply overall acceleration, as discussed.)

We now review the best measurements of  $t_0$ ,  $H_0$  and  $\Omega_m$ . Systematically combining the results of distinct measurements using different techniques, each with different statistical and systematic errors, is challenging. However, our discussion at least allows a reasonable guess at current global best values and uncertainties for these quantities. Examples of more sophisticated Bayesian analysis are given in [12].

$t_0$ : Our approach is made possible by relatively recent measurements of  $t_0$  with unprecedented accuracy. In the past, estimates of  $t_0$  have been made by either using model-dependent estimates for the ages of globular clusters or through nuclear cosmochronometry. The former method has traditionally suffered from the unknown role of convection and its effects on the lifetimes of low mass/low metallicity stars. Krauss and Chaboyer [13] performed a thorough Monte Carlo analysis that includes these uncertainties, to arrive at a firm lower limit of 11.2 Gyr for  $t_0$ . However,  $t_0$  as large as 15 Gyr is still allowable. Using Thorium cosmochronometry, Sneden and Cowan [14] also find a lower limit of 11 Gyr for  $t_0$  but acknowledge that lower limit could range upwards by another 3–4 Gyr. For the reasons cited, we do not use these methods or observations in constructing our argument for the most probable value of  $t_0$ .

Improvements in the precision of measuring  $t_0$  have utilized the white dwarf cooling curve and Hubble Space Telescope measurements of the halo globular cluster M4. Measurements by Hansen et al. (2002) [15] report a value of  $12.7 \pm 0.7$  Gyr. Hansen et al. (2004) [16] update this age to  $12.1 \pm 0.9$  Gyr.

The major source of systematic error in this analysis involves estimating the lag time between the age of the Universe and the formation of globular clusters. Numerical simulations of the Milky Way and its globular cluster system by Kravtsov and Gnedin (2005) [17] indicate that the peak formation of Globular Clusters occurs at  $z = 3\text{--}5$ . Using a mean formation redshift of  $z = 4$  implies that Globular Clusters formed at 1.2 Gyr after the onset of the Big Bang. This then leads to a lower limit of  $t_0 = 12.4$  Gyr and a mean value of  $t_0 = 13.3^{+1.1}_{-0.9}$  Gyr.

$H_0$ : For decades, measurements of  $H_0$  were plagued by noise and biased samples. Today, however, there is good reason to believe that we have a relatively precise measure for this parameter as well. The Hubble Space Telescope Key Project for determining the Cepheid Zero Point and subsequent distance determinations to nearby galaxies using the Cepheid Period-luminosity relationship have returned a value of  $72 \pm 3$  km/s/Mpc [18]. The major source of systematic uncertainty in that measurement lies in the distance to the Large Magellanic Cloud (LMC), to which the zeropoint of the Cepheid Luminosity scale is anchored. Freedman and Madore [19] quote a total systematic error of  $\pm 7$  km/s/Mpc, but recent improved distance estimates for the LMC (e.g., Benedict et al. (2002) [20] and Sebo et al. [21]) have served to lower this systematic error down to  $\pm 4$  km/s/Mpc (see Ngeow and Kangur (2006) [22]). Moreover, confidence in the precision of  $H_0$ , as anchored by the LMC distance, is reinforced by recent measurements that are completely independent of the distance to the LMC. In the past, these kinds of measurements were also available but they had sufficiently large random error that precluded them from providing meaningful constraints on the value of  $H_0$  as determined from traditional distance scale ladder techniques. The new observations are:

(1) Using a sample of 38 X-ray clusters in combination with the Sunyaev–Zeldovich effect, Bonamente et al. (2006) [23] derive a value of  $H_0 = 77.6 \pm 5$  km/s/Mpc. While there may be systematics associated with the non-spherical shape of clusters, their sample size is sufficiently large (and much larger than past samples) that this problem is removed by averaging.

(2) Wang et al. [24] have examined a sample of 109 SN of type Ia and have discovered important new corrections for metallicity and absorption (by dust) in determining SN Ia peak luminosity. This recalibration leads to  $H_0 = 72 \pm 6$  km/s/Mpc. An independent treatment of SN Ia has been compiled by Riess et al. (2005) [25] which yields a value of  $H_0 = 73 \pm 4$  km/s/Mpc with possible systematic error of  $\pm 5$  km/s/Mpc.

(3) Koopmans et al. [26] perform a detailed analysis of a gravitational lens system (from which a direct determination of the distance can be determined using a model mass distribution of the lens) to find  $H_0 = 75 \pm 6.5$  km/s/Mpc.

Averaging these 5 different results together formally leads to  $74 \pm 2.5$  km/s/Mpc (error in the mean). Direct averaging is crude, but gives a characterization of the uncertainty. Averaging over systematic errors as well, we assume  $H_0 = 74 \pm 5$  km/s/Mpc in further analysis. In contrast, one could use only method 1 and 3 above (as they completely circumvent the LMC distance problem) to obtain  $76 \pm 6$  as the relevant range.

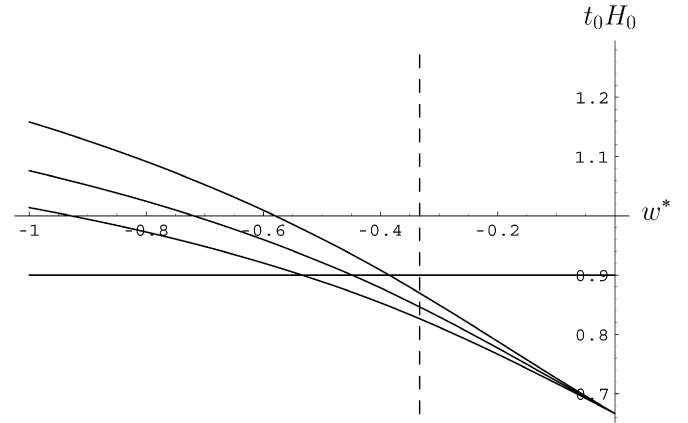


Fig. 1. Curves in the  $w$ - $t_0 H_0$  plane, each of which is an *upper* bound on  $t_0 H_0$ , for  $\Omega_m = 0.25, 0.20, 0.15$ . The allowed region is between the top and bottom curves, and above the horizontal line  $t_0 H_0 = 0.9$ . This requires  $w$  less than  $-1/3$ .

$\Omega_m$ : In contrast,  $\Omega_m$  remains the most weakly constrained cosmological observable. There are two reliable methods of measurement: dynamical determinations based on infall to clusters of galaxies and/or the nature of large scale structure (e.g., Bothun et al. [27]) or by fitting the Hubble diagram to distant objects. In the first case, an unbiased and fairly large sample is needed for precision; in the second case, accurate distance measurements of intermediate redshift galaxies are required, and such measurements are ultimately based on the supernova luminosity scale. In principle,  $\Omega_m$  is highly constrained by the multi-parameter maximum likelihood fit to the WMAP data; but this is an indirect determination of  $\Omega_m$  (as well as  $t_0$ ). In the spirit of this analysis, we seek to use values of  $\Omega_m$  that have been directly determined.

Note, though, that  $\Omega_m$  is now usually determined by assuming a flat Universe as a prior constraint. For instance, a recent accurate determination of  $\Omega_m$  results from analysis of the power spectrum of galaxy clustering. Assuming a flat Universe, Sanchez et al. (2006) [28] find  $\Omega_m = 0.237 \pm 0.02$ . In addition, Mohayee and Tully (2005) [29] revisit the peculiar velocities of galaxies in the Local Supercluster to derive  $\Omega_m = 0.22 \pm 0.02$ . Schindler (2002) [30] summarizes all techniques to determine  $\Omega_m$  (including the more unreliable approaches such as the X-ray cluster luminosity function, weak gravitational lensing, or galaxy cluster evolution). That summary yields a modal value of  $\Omega_m = 0.3$  (which is likely a realistic upper limit given the WMAP model) but also shows that most large scale structure studies yield values of  $\Omega_m$  in the range 0.20–0.25 (which is consistent with the work done in the 1980s). Averaging together the Sanchez et al. and Mohayee and Tully studies produces a well constrained value of  $\Omega_m = 0.23 \pm 0.02$ . For discussion below we take a conservatively large range for  $\Omega_m$ , assuming 0.15–0.25 to be a one standard deviation range about the central value.

**Results:** In Fig. 1, we plot  $I(\Omega_m, w)$  for  $\Omega_m = 0.15, 0.20$  and 0.25.  $\Omega_m = 0.15$  corresponds to the curve with the largest values of  $t_0 H_0$ . Taking  $t_0 = 12.4$  Gyr and  $H_0 = 69$  km/s/Mpc, which are each one standard deviation below the favored (cen-

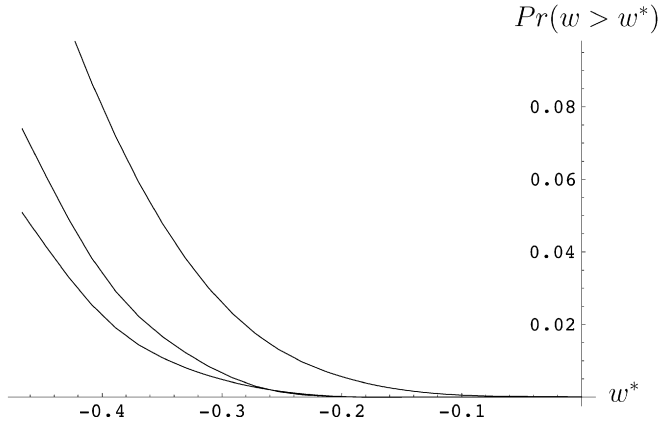


Fig. 2. Probability that  $w$  was always greater than  $w^*$  for a range of  $w^*$  and various cuts on  $t_0$  and  $\Omega_m$ . See text for details.

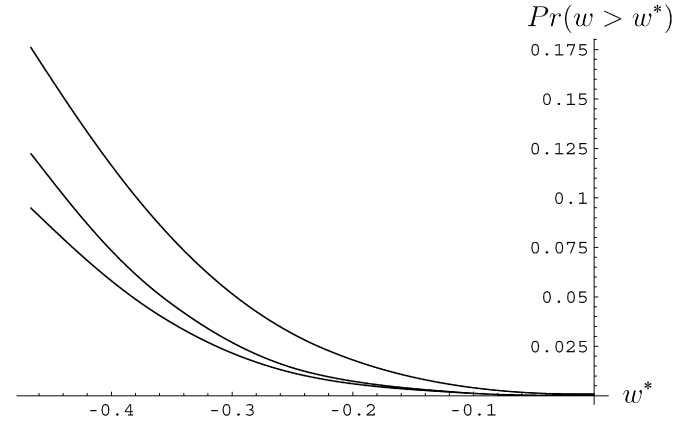


Fig. 3. Same as Fig. 2 except with larger Hubble uncertainty:  $H_0 = 74 \pm 7$  km/s/Mpc.

tral) values in our assumed error model, we obtain  $t_0 H_0 = 0.9$ , which corresponds to the grey horizontal line in the figure. The implications can be read directly from the figure. If  $w$  was always greater than  $-1/3$ , then some or all of our parameters must be well below their central values.

From Fig. 1, we see that taking  $t_0$ ,  $H_0$  and  $\Omega_m$  to each be one standard deviation below their central value (so,  $t_0 = 12.4$  Gyr,  $H_0 = 69$  km/s/Mpc and  $\Omega_m = 0.15$ ), an epoch with  $w < -0.4$  or so is required, which is just negative enough to imply acceleration ( $\dot{R} > 0$ ). Taking  $t_0 = 12.4$  Gyr and  $\Omega_m = 0.15$ , one would have to, e.g., push  $H_0$  below 67 km/s/Mpc to have  $w > -1/3$ , and below 50 km/s/Mpc to have  $w > 0$  (no negative pressure).

We compute the likelihood of no epoch with  $w < w^*$  (for given  $w^*$ ) as follows. First, we assume uncorrelated Gaussian errors in all three parameters:  $t_0 = 13.3 \pm 1$  Gyr,  $H_0 = 74 \pm 5$  km/s/Mpc and  $\Omega_m = 0.2 \pm 0.05$  (all one standard deviation). That is, we assume that the probability distribution for the actual value each of parameter is normal, with maximum at the central value and standard deviation given by the error estimate. We then compute, for a particular value of  $w^*$ , the total probability that the parameters take on values for which inequality (6) is satisfied. In practice, this was done using Monte Carlo.

The results are displayed in Fig. 2 (top curve). Using this error model the probability of no epoch with  $w < -1/3$  is less than 4 percent. This is an *overestimate* of the likelihood, since the model allows values of, e.g.,  $t_0$  which are much too low:  $t_0 = 12.4$  Gyr is more plausibly interpreted a strict minimum than minus one standard deviation from the central value. Modifying the error model so that values of  $t_0 < 12.4$  Gyr are not allowed reduces the likelihood of no epoch with  $w < -1/3$  to about 1.3 percent. This is represented by the middle curve in Fig. 2. Adding a similar constraint that  $\Omega_m > 0.15$  leads to the lowest curve in the figure, and a likelihood of no epoch with  $w < -1/3$  of about 0.8 percent. Fig. 3 is identical to Fig. 2 except that we have increased the one standard deviation error for  $H_0$  to  $\pm 7$  km/s/Mpc; the existence of dark energy is still strongly favored.

We conclude that, unless systematic errors are significantly larger than currently recognized, best measurements of the age of the Universe  $t_0$ , the Hubble parameter  $H_0$  and the matter frac-

tion  $\Omega_m$  strongly favor the existence of a repulsive dominant energy component, also known as dark energy. These observations are independent of type Ia supernova surveys: specifically, they are not sensitive to uncertainties [7–9] which affect the direct measurement of the distance-redshift relation at large  $z$ .

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